Week 4 PYQs
Machine Learning Foundations
May 22 - Sep 25

Note

1. If no questions are listed in that term Quiz, it indicates that the corresponding quiz did not include questions from that week.

2. The answer key may contain minor inaccuracies, as answers were taken directly from the original quiz/exam papers. Some of these might have been corrected later.

3. For any issues or suggestions regarding this document, feel free to contact:
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Quiz 1 May 22

1. $f(x_1, x_2, x_3) = \frac{x_1+x_2+x_3}{3}$ is used as encoder and $g(u) = [u, 2u, 3u]$ as decoder for dataset: $\{(1, 2, 3), (2, 3, 4)\}$. Compute reconstruction error.

   [Ans: 24-25]

2. Find the linear approximation L of $f(x_1, x_2) = 2x_1^2 + 4x_2^2$ around $(1, 1)$ and compute $f(1.1, 1.1) - L(1.1, 1.1)$.

   [Ans: 0.2-0.3]

3. The eigenvalues of the matrix
   $$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & 0 \\ -2 & 2 & 1 \end{bmatrix}$$
   are a, b, c in increasing order. Then the value of $a + b$ is:

   [Ans: 0]

4. If $A = PDP^{-1}$ where
   $$P = \begin{bmatrix} 1 & 2 \\ 1 & -5 \end{bmatrix}, \quad D = \begin{bmatrix} -1 & 0 \\ 0 & 6 \end{bmatrix}$$
   Compute A^4 and sum of all its elements.

   [Ans: 1297]

5. Find α, β such that
   $$A = \begin{bmatrix} 2 & \alpha \\ 2 & \beta \end{bmatrix}$$
   has eigenvalues -1 and -3. Compute $2\alpha + \beta$.

   [Ans: -21]

6. Projection of $(2, -1, 3)^T$ onto $(-1, 3, 1)^T$. What is squared length?

   (a) 0.8686
   (b) 0.3636
   (c) 1.0363
   (d) 1.3636

   [Ans: b]

7. Dataset:
   $$(1, 2), (2, 1), (1, 3), (3, 4)$$
   Find least squares best fit line.

(a) $\hat{y} = 0.412x + 0.215$

(b) $\hat{y} = 1.335x + 2.245$

(c) $\hat{y} = 0.545x + 1.545$

(d) $\hat{y} = 1.713x + 1.175$

(e) $\hat{y} = 2.225x + 3.665$

[Ans:c]

Quiz 2 May 22

ET May 22

1. Suppose that eigenvalues of matrix A are 1, 0.5, 0.25. The determinant of $(A^{-1})^T$ is *NOTE: Enter your answer to the nearest integer.* [Ans: 8]

2. Consider the following matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$. Which of the following are the eigenvectors of this matrix?

   (a) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

   (d) $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

   [Ans: 1, 2, 3]

Quiz 1 Sep 22

1. The ML team in a movie production company wanted to predict whether a movie will earn 100 cr or not using some classification models. Therefore, the team collected presence/absence of various factors $x = [x_1, x_2, x_3]$ from the movies in the past. The data and labels are:

x	y
$[0, 1, 1]$	0
$[1, 0, 1]$	1
$[1, 0, 0]$	1
$[1, 1, 1]$	1
$[1, 1, 0]$	1

   Compute the loss of the model:

$$u(z) = \begin{cases} 1, & z \geq 0 \\ 0, & \text{otherwise} \end{cases} \quad \text{where } z = 0.5x_1 - 0.8x_2 + 0.4x_3$$

   [Ans: 0.18–0.24]

2. Find the linear approximation L of $f(x, y) = 3x^3 + 4y^2 + 10$ around $(1, 1)$ and compute $f(1.1, 0.9) - L(1.1, 0.9)$.

   [Ans: 0.10-0.16]

3. If the second degree polynomial that fits the below is $ax^2 + bx + c$, then what is the value of $a + b + c$?

x	y
1	2
2	3
3	7

   [Ans:2]

4. Which of the following are eigenvectors of

$$\begin{bmatrix} 5 & 2 \\ 3 & 6 \end{bmatrix}$$

   (a) $(1, -1)^T$

   (b) $(1, 1)^T$

   (c) $(1, 1.5)^T$

(d) $(2,3)^T$

[Ans: a, c, d]

5. What is the length of orthogonal projection of vector $(2,2,3)^T$ onto a line through $(2,3,6)^T$? Round the answer to nearest integer.

[Ans: 4]

6. If $A = PDP^{-1}$ where

$$P = \begin{bmatrix} 1 & 2 \\ 1 & -5 \end{bmatrix}, \quad D = \begin{bmatrix} -1 & 0 \\ 0 & 6 \end{bmatrix},$$

then what is the summation of all the diagonal elements of A^4?

[Ans: 1297]

7. Find the values of α and β for which the matrix

$$A = \begin{bmatrix} 2 & \alpha \\ 2 & \beta \end{bmatrix}$$

has eigenvalues equal to -1 and -3. Enter your answer as $2\alpha + \beta$.

[Ans: -21]

8. In a linear regression problem $X\theta = y$, with features

$$X = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 1 & 5 \end{bmatrix}$$

and

$$y = (16, 9, 18)^T,$$

what is the weight vector θ?

(a) $(-3,2,5)^T$
(b) $(3,2,5)^T$
(c) $(-1,1,6)^T$
(d) $(-3,5,2)^T$

[Ans: a]

Quiz 2 Sep 22

ET Sep 22

Quiz 1 Jan 23

1. Let P be the matrix that projects vectors in $\mathbb{R}^3$ onto the subspace spanned by $[1, 2, 3]^T$. Which of the following are eigenvectors of P corresponding to eigenvalue 0?

   (a) $[1, -2, 2]$
   (b) $[2, 2, -1]$
   (c) $[-2, 1, 0]$
   (d) $[-3, 0, 1]$

   [Ans: c, d]

2. A quadratic approximation of the function $f(x) = \sin^2 x$ at $x = \frac{\pi}{4}$ is:

   (a) $Q(x) = \frac{1}{2} + (x - \frac{\pi}{4}) + (x - \frac{\pi}{4})^2$
   (b) $Q(x) = \frac{1}{2} + (x - \frac{\pi}{4})$
   (c) $Q(x) = \frac{\pi}{4} + 2(x - \frac{\pi}{4}) + \frac{1}{2}(x - \frac{\pi}{4})^2$
   (d) $Q(x) = \frac{1}{2} + 2(x - \frac{\pi}{4}) + \frac{1}{2}(x - \frac{\pi}{4})^2$

   [Ans: b]

3. The characteristic polynomial of a matrix with eigenvalues $1, 2, 3, 4$ is:

   (a) $\lambda^4 + 10\lambda^3 + 35\lambda^2 + 50\lambda + 24$
   (b) $\lambda^4 + 10\lambda^3 - 35\lambda^2 + 50\lambda - 24$
   (c) $2\lambda^4 - 20\lambda^3 + 35\lambda^2 - 100\lambda + 48$
   (d) $\lambda^4 - 10\lambda^3 + 35\lambda^2 - 50\lambda + 24$

   [Ans: d]

4. If projection of vector A of length 10 units onto vector B of length 8 units has magnitude 5 units, then length of projection of vector B onto A is?

   [Ans: 4]

Quiz 2 Jan 23

ET Jan 23

1. Which of the following statements are correct?

   (a) If 0 is the eigenvalue of a matrix A, then A can't be invertible.

   (b) If all the eigenvalues of a matrix A are 0, then A is a zero matrix.

   (c) If 3 is the eigenvalue of matrix A then 9 is the eigenvalue of A^2.

   (d) Every 2×2 matrix has atleast one real eigenvector.

[Ans: 1, 3]

Quiz 1 May 23

1. Which of the following matrices have eigenvectors as

$$\begin{bmatrix}2\\1\\2\end{bmatrix}, \begin{bmatrix}1\\-2\\0\end{bmatrix}, \begin{bmatrix}1\\0\\-1\end{bmatrix}?$$

(a) $\begin{bmatrix}3 & 2 & 4\\2 & 0 & 2\\4 & 2 & 3\end{bmatrix}$

(b) $\begin{bmatrix}6 & 4 & 8\\4 & 0 & 4\\8 & 4 & 6\end{bmatrix}$

(c) $\begin{bmatrix}2 & 1 & 4\\4 & 0 & 4\\8 & 4 & 6\end{bmatrix}$

(d) $\begin{bmatrix}3 & 1 & 0\\2 & 0 & 2\\4 & 2 & 3\end{bmatrix}$

[Ans: a, b]

2. Consider the solution of the linear regression problem ($Ax = b$) using least squares method, where

$$A = \begin{bmatrix}1 & 5 & 5\\2 & 3 & 5\\0 & 7 & 1\end{bmatrix}, \quad b = \begin{bmatrix}7\\8\\4\end{bmatrix}$$

Based on this data, which of the following statements is/are true?

(a) The error value is zero as we can get an x such that Ax is exactly equal to b.

(b) The error value is non-zero as we cannot get an x such that Ax is exactly equal to b.

(c) Least squares solution for x is $\begin{bmatrix}2\\0.5\\0.5\end{bmatrix}$

(d) Least squares solution for x is $\begin{bmatrix}1\\0.6\\0.4\end{bmatrix}$

[Ans: a, c]

3. Let $f(x_1, x_2, x_3) = \frac{x_1+x_2+x_3}{2}$ and $g(u) = [u, u, u]$. Dataset: $[1, 2, 3], [2, 4, 6], [3, 6, 9], [4, 8, 12]$. Compute reconstruction error.

[Ans: 37–38]

Quiz 2 May 23

ET May 23

1. Let $A = \begin{bmatrix} 3 & -2 \\ 5 & -4 \end{bmatrix}$, Find the trace of A^{10}. [Ans: 1025]

Quiz 1 Sep 23

1. Let
$$A = \begin{bmatrix} 8 & 3 \\ 2 & 7 \end{bmatrix}.$$
Find a matrix P such that $P^{-1}AP$ is diagonal.

   (a) $\begin{bmatrix} 1 & 1 \\ 2 & -3 \end{bmatrix}$

   (b) $\begin{bmatrix} 10 & 0 \\ 0 & 5 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$

   (d) $\begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}$

   [Ans: d]

2. Consider a 2×2 matrix with eigenvectors $[1, 1]^T$ and $[1, 2]^T$ and eigenvalues $2, 3$. Find sum of squares of all entries.

   [Ans: 22]

3. **Comprehension:**

   Consider a subspace $U \subseteq \mathbb{R}^3$ and vector $b \in \mathbb{R}^3$ where
$$U = \text{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \right\}, \quad b = \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix}$$

   (a) Find projection of b onto U.

   i. $\begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$

   ii. $\begin{bmatrix} -1 \\ -1 \\ -2 \end{bmatrix}$

   iii. $\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$

   iv. $\begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$

[Ans: ii]

(b) Find length of error vector $e = b - p$.

[Ans: 3.70–3.78]

Quiz 2 Sep 23

ET Sep 23

1. Which among the following statements are correct?

   (a) The eigenvalues of a matrix are on its main diagonal.

   (b) Each eigenvector of an $n \times n$ matrix A is also an eigenvector of A^2.

   (c) Two eigenvectors corresponding to the same eigenvalues are always linearly dependent.

   (d) If v_1 and v_2 are linearly independent eigenvectors, then they correspond to distinct eigenvalues.

   [Ans: 2]

Quiz 1 Jan 24

1. Let
$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}.$$
Find a matrix P such that $P^{-1}AP$ is diagonal.

   (a) $\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$

   (d) $\begin{bmatrix} -2 & -2 \\ 0 & 0 \end{bmatrix}$

   [Ans: b]

2. Find the best fit line for the dataset given below using least squares method.

x	0	2	3	6	8
y	−2	5	3	1	−3

   (a) $\hat{y} = 0.6x - 3.2$

   (b) $\hat{y} = -0.6x + 3.2$

   (c) $\hat{y} = -x + 2.8$

   (d) $\hat{y} = -0.4x + 2.8$

   [Ans: b]

3. Consider a matrix
$$B = A^4 - 3A - 2I, \quad \text{where } A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}.$$
Which of the following options is/are true?

   (a) Eigenvalues of A are $1, 2$, and 3.

   (b) Eigenvalues of A are $1, 2$, and 2.

   (c) Eigenvalues of B are $2, 20$, and 78.

   (d) Matrix B is diagonalizable.

[Ans: a, d]

4. Let A and B be symmetric matrices such that $AB = BA$. Then which of the following options is/are true?

   (a) AB is a symmetric matrix.

   (b) AB is orthogonally diagonalizable.

   (c) BA has real eigenvalues.

   (d) AB is not a symmetric matrix.

[Ans: a, b, c]

Quiz 2 Jan 24

ET Jan 24

Quiz 1 May 24

1. Let
$$A = \begin{pmatrix} 2 & 1 & 1 \\ a & 3 & 2 \\ 3 & b & c \end{pmatrix}$$
be a 3×3 matrix.

Let α be the eigenvalue corresponding to
$$v_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$
and β be the eigenvalue corresponding to
$$v_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

(a) Find $\alpha + 2\beta$. [Ans: 3]

(b) Which of the following are true?

i. Eigenvectors v_1 and v_2 are linearly independent

ii. Eigenvectors v_1 and v_2 are orthogonal

iii. A third eigenvector of A is $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

iv. a+b+c=0 [Ans: a,c,d]

2. Let
$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

(a) Which of the following are true?

i. A is symmetric matrix

ii. A is diagonal matrix

iii. A is orthogonally diagonalizable

iv. Eigenvalues are $-1, -1, 2$ [Ans: a,c,d]

(b) Let v_1, v_2 be eigenvectors corresponding to distinct eigenvalues. Let $B = [v_1\ v_2]$.

i. Null space of B is zero space

ii. Column space of B is $\mathbb{R}^3$

iii. Column space of B represents a plane in $\mathbb{R}^3$

iv. Null space represents a line in $\mathbb{R}^3$

[Ans: a,c]

3. Let P be projection matrix for vector $a = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

   (a) P is orthogonally diagonalizable

   (b) $P^4 = P$

   (c) Projection of $b = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ onto a is $-\frac{1}{5}a$

   (d) Rank$(P) = 2$

[Ans: a, b,c]

4. Find best fit line for:

x	0	2	3	5
y	1	2	4	3

   (a) $\hat{y} = 1.34 + 0.46x$

   (b) $\hat{y} = 0.46 + 1.34x$

   (c) $\hat{y} = 1.34 - 0.46x$

   (d) $\hat{y} = 2.16 + 0.74x$

[Ans: a]

Quiz 2 May 24

ET May 24

1. Consider a square matrix A of order 3 such that trace(A) = 3 and det(A) = 2 If 1 is an eigenvalue of A, then which of the following options is/are true?

   (a) Matrix A is symmetric matrix.

   (b) Matrix A is Hermitian matrix.

   (c) $1 + i$ is an eigenvalue of A.

   (d) $2 - i$ is an eigenvalue of A.

   (e) Matrix A is diagonalizable.

[Ans: 3, 5]

Quiz 1 Sep 24

1. Consider a matrix

$$B = A^4 - 3A - 2I, \quad \text{where } A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}.$$

Which of the following options is/are true?

(a) B has eigenvalues $-4, 8, 70$.

(b) A has eigenvalues $1, 16, 81$.

(c) B is an upper triangular matrix.

(d) B is a lower triangular matrix.

[Ans: a, c]

2. Consider a matrix

$$A = \begin{bmatrix} 6 & 3 \\ -2 & 1 \end{bmatrix}$$

and a set $S = \{v_1 = (1, -1), v_2 = (1, 1)\}$. Which of the following options is/are correct?

(a) S is the set of eigenvectors of A.

(b) Only v_1 is an eigenvector of A.

(c) $A^{50} = \begin{bmatrix} 1 & 3 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 3^{50} & 0 \\ 0 & 4^{50} \end{bmatrix} \begin{bmatrix} -2 & -3 \\ 1 & 1 \end{bmatrix}$

(d) $A^{50} = \begin{bmatrix} 3 & 1 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} 3^{50} & 0 \\ 0 & 4^{50} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -2 & -3 \end{bmatrix}$

[Ans: b, c]

3. Consider two vectors

$$a = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}.$$

Let P be the projection matrix that projects a vector onto the line through the vector a. Which of the following options is/are true?

(a) P is symmetric matrix.

(b) Projection of b onto a is $\frac{1}{4}(-a)$.

(c) P is rank one matrix.

(d) Plane $x = y$ is orthogonal to a.

[Ans: a, c, d]

4. Which of the following options is/are correct for the following dataset?

$$\begin{array}{c|ccc} x & 1 & 1 & -1 \\ y & 1 & 1 & 1 \end{array}$$

   (a) The best-fit line using least squares method is $\hat{y} = 1$.

   (b) The best-fit line is $\hat{y} = -1$.

   (c) The best-fit line is $\hat{y} = x - 1$.

   (d) The best-fit line is $\hat{y} = x + 1$.

   (e) Error vector is $\begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$.

   (f) Error vector is $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

[Ans: a, f]

5. Let

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 6 \end{bmatrix}.$$

   Is A diagonalizable?

   (a) Yes

   (b) No

   (c) Insufficient information

[Ans:a]

Quiz 2 Sep 24

1. Which of the following statements is true about the given matrix M?

$$M = \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}$$

   (a) M is an invertible matrix.

   (b) M is a singular matrix.

   (c) The determinant of M is 4.

   (d) M is diagonalizable.

[Ans: 2, 4]

ET Sep 24

1. Which of the following options is/are true?

   (a) If $det(A) = 0$ for a given matrix A, then the matrix A has 0 as an eigenvalue.

   (b) For a given matrix $A_{m\times n}$ the non-zero eigenvalues of matrices A^TA and AA^T are the same.

   (c) If $det(A) \neq 0$ for a given matrix A, then nullity of the matrix A is 0.

   (d) If a matrix $A_{n\times n}$ has only non-zero eigenvalues, then the rank of the matrix A is $n - 1$.

[Ans: 1, 2, 3]

Quiz 1 Jan 25

1. Consider the matrix
$$M = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}.$$
Which of the following options is true?

   (a) Eigenvectors corresponding to two different eigenvalues are orthogonal.

   (b) The matrix M is not diagonalizable.

   (c) The eigenvalues of M are 4 and 1.

   (d) The rank of M is 1.

[Ans: d]

2. Which of the following is the best second-order polynomial that fits the dataset by least squares?

x	1	0	-1
y	-1	1	1

   (a) $2 - \frac{1}{2}x + \frac{5}{2}x^2$

   (b) $2 - \frac{1}{2}x - \frac{5}{2}x^2$

   (c) $2 + \frac{1}{2}x - \frac{5}{2}x^2$

   (d) $2 + \frac{1}{2}x + \frac{5}{2}x^2$

[Ans: b]

3. Which of the following options is/are true?

   (a) $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ is orthogonal to $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

   (b) For a matrix A, column space vectors are perpendicular to null space vectors.

   (c) Length of $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ is 3.

   (d) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ are orthogonal.

[Ans: a, d]

4. Which of the following options is/are true when the vector $\begin{bmatrix}1\\2\\1\end{bmatrix}$ is projected onto $\begin{bmatrix}1\\-1\\0\end{bmatrix}$?

   (a) Projection matrix is $\frac{1}{2}\begin{bmatrix}-1 & -1 & 0\\1 & 1 & 0\\0 & 0 & 0\end{bmatrix}$

   (b) Projection matrix is $\frac{1}{2}\begin{bmatrix}1 & -1 & 0\\-1 & 1 & 0\\0 & 0 & 0\end{bmatrix}$

   (c) Projected vector is $\frac{1}{2}\begin{bmatrix}-1\\1\\0\end{bmatrix}$

[Ans: b, c]

5. Consider a 3×3 matrix B given by

$$B = (-1)\begin{bmatrix}-4 & 3 & 0\\-2 & 1 & 0\\-1 & 1 & -1\end{bmatrix}.$$

   Based on the above data, answer the given subquestions.

   (a) If B is diagonalizable, what is the invertible matrix P such that $B = PDP^{-1}$?

   i. $\begin{bmatrix}4 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

   ii. $\begin{bmatrix}3 & 2 & 1\\2 & 1 & 1\\1 & 0 & 1\end{bmatrix}$

   iii. $\begin{bmatrix}1 & 1 & 1\\2 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

   iv. $\begin{bmatrix}3 & 1 & 0\\2 & 1 & 0\\1 & 0 & 1\end{bmatrix}$

[Ans: d]

(b) If B is diagonalizable, which of the following is a diagonal matrix D such that $B = PDP^{-1}$? Use the matrix P obtained in the previous question.

i. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

ii. $\begin{bmatrix} 4 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

iii. $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

iv. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

v. B is not diagonalizable

[Ans: a]

Quiz 2 Jan 25

ET Jan 25

Quiz 1 May 25

1. Consider a matrix
$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$
Which of the following options is/are true?

   (a) 1 is an eigenvalue of A.

   (b) 2 is an eigenvalue of A.

   (c) $\begin{bmatrix} -3 \\ -1 \\ 2 \end{bmatrix}$ is an eigenvector of A.

   (d) $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ is an eigenvector of A.

[Ans: a, b, c]

2. Consider a matrix
$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad A = PDP^{-1}.$$
Which of the following options is/are true?

   (a) A is an orthogonally diagonalizable matrix.

   (b) $P = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

   (c) $P = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$

   (d) $P^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 0 & -1 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$

[Ans: a, b]

3. Let $\hat{y}$ be the regression line for the dataset:

x	0	2	3
y	-2	5	4

Which of the following is/are true?

(a) $\hat{y} = \frac{1}{14}(13x - 19)$

(b) $\hat{y} = \frac{1}{14}(31x - 91)$

(c) If $x = 11$, predicted $y = 23$

(d) If $x = 7$, predicted $y = \frac{99}{7}$

[Ans: c, d]

Quiz 2 May 25

ET May 25

1. Consider the vectors $v_1 = \begin{bmatrix} 3 \\ 1 \\ 1 \\ -2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ -1 \\ -1 \\ 1 \end{bmatrix}$ and $v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 3 \end{bmatrix}$. Let $S = span(v_1)$. Which of the following options is/are true?

   (a) The projection of v_2 on to S is $\frac{2}{15}v_1$

   (b) The point in S closest to v_2 is $\frac{-2}{15}v_1$.

   (c) v_3 is orthogonal to v_1

   (d) v_3 is not orthogonal to v_2

[Ans: 1, 3, 4]

Quiz 1 Sep 25

1. Consider the dataset $(x, y) : (-1, 2), (0, 1), (1, 2)$. Perform polynomial regression of degree 2 by fitting a quadratic model of the form $y = \theta_0 + \theta_1 x + \theta_2 x^2$. Using the method of least squares, find the estimates of θ_0, θ_1 and θ_2.

   (a) $y = x^2 + 1$

   (b) $y = x^2$

   (c) $y = x^2 - 1$

   (d) $y = 2x$

   [Ans:a]

2. Does there exist a matrix A of shape 10×10 such that every non-zero vector in $\mathbb{R}^{10}$ is an eigenvector with eigenvalue 2? If yes, compute $\det(A)$, else enter -1.

   [Ans: 1024]

3. If the sum of the columns of a square matrix A is the zero vector, then λ is an eigenvalue of A. What is λ?

   [Ans: 0]

4. Which of the following is/are true?

   (a) A is an orthogonal matrix

   (b) A is invertible

   (c) A is symmetric

   [Ans: a,b]

5. Which of the following is true?

   (a) A does not have real eigenvalues

   (b) A has two distinct real eigenvalues

   (c) A has two real eigenvalues, not necessarily distinct

   [Ans: a]

6. If a square matrix A is diagonalizable, then A is invertible.

   (a) True

   (b) False

[Ans: b]

7. The eigenvalues of a real symmetric matrix are real.

   (a) True
   (b) False

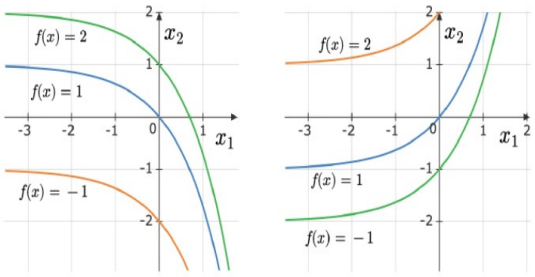
[Ans: a]

8. A square matrix A is diagonalizable iff it has n distinct eigenvalues.

   (a) True
   (b) False

[Ans: b]

9. Which contour plot is correct?



   (a) (A)
   (b) (B)

[Ans: a]

Quiz 2 Sep 25

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